HAS OFFSHORE OIL PRODUCTION BECOME SAFER?

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ABSTRACT: In what follows we examine the hypothesis that there has been no improvement in the offshore oil production safety vs the hypothesis that there has been a gradual improvement.

Our analysis will show that the second hypothesis is much better supported by the available data.

INTRODUCTION

One assumption concerning oilspills due to activities on offshore platforms is that they follow a Poisson distribution with parameter proportional to the oil produced. Of particular interest are spills big enough to create environmental hazards by themselves, and attention has focused on spills of 1,000 barrels or more. In what follows the term spill, unless otherwise qualified, will refer to spills of 1,000 barrels or more that originate from oil production platforms on the U.S. Outer Continental Shelf.

A look at the U.S. offshore oil production from 1964 to December 1979 and at the spill data shows that:

a. Production rose from 1964 to 1971 and has been declining ever since (Table I).

b. The estimated amount of oil produced between two successive spills shows a tendency to increase with time (Table II).

The question is then:

Is there any statistical evidence that we are not dealing with a Poisson process whose parameter is proportional to the oil produced?

or equivalently:

Is there any reason to believe that the estimated amounts of oil produced between successive accidents $(x_1, x_2, .., x_i, .)$ are not realizations of independent and identically distributed exponential variables X_1, X_2, ..., X_i, ... ?

To test the latter, we applied some distribution free tests to the observed variables and we applied a second set of tests by constructing variables whose distributions are free of λ (the ratio of the Poisson parameter over the oil produced). The following results were obtained:

(A) Runs, runs-up, and runs-down. The signs of the differences of our observations are:

+ + + - + + + +

Let run-up be a maximal uninterrupted sequence of "+"s and run-down a maximal uninterrupted sequence of "-"s. Then, for nine observations we have:

(1) The probability of three runs or fewer is 2.57%. (Bradley, 1968, Table X)

(2) The probability that the longest run will have a length of 4 or more is 7.18%. (Olmstead, 1964, Table 2)

(3) The expected number of runs-up of length 4 or more is 0.0361. (Bradley, 1968, p. 274)

(4) The probability that the longest run-up will have length 4 or more is therefore greater than 3.59% but less than 3.61%.

Indeed, any given sequence of signs may contain runs-up, runs-down or runs-up, and runs-down of a given length.

Thus:

$$\text{Prob (longest run-up} \geq k) \geq \text{Prob (longest run} \geq k)/2$$

(giving in our case)

$$\text{Prob (longest run-up} \geq 4) \geq 7.18/2 = 3.59$$

On the other hand, the expected number of runs-up of length k or more exceeds the probability of having at least one run-up of length k or more since the former equals:

$$\Sigma \ell P \text{ (exactly } \ell \text{ runs-up of length k or more)}$$

while the latter equals:

$$\Sigma_{\ell} P \text{ (exactly } \ell \text{ runs of length k or more)}$$

(B) Kendall's test for correlation:

Given n observations $x_1, x_2 .. x_n$, Kendall (1955) defines T as the number of pairs of numbers in the sequence in which the smaller observation precedes the bigger one and I, as the number of pairs in the sequence in which the bigger observation precedes the smaller one. Kendall's statistic is $S = T - I$ and is a measure of the tendency of our observations to form a monotonic sequence.

In our case there are nine observations, no ties, $I = 3$, $T = 33$, and $S = 30$.

From Bradley (1968, Table XI) we find that:

Prob $(S \geq 30 \mid n = 9) < 0.5\%$.

(C) Hotelling and Pabst's test.

Given n observations $x_1, x_2 .. x_n$, we define rank (x_i) to be the number of x_j's, $j = 1,2..,i,...n$, which do not exceed x_j. The statistic

$$D = \sum_{i=1}^{n} [i - \text{rank}(x_i)]^2$$

is a measure of the tendency of our observations to form a monotonic sequence, with small D values indicating increasing sequence, and large D values indicating decreasing sequences. In our case we have:

x	31.3	55.9	103.1	712.9	43.5	270.5	272.5	804.9	2054.3
rank(x)	1	3	4	7	2	5	6	8	9

and we compute:

$$D = 0 + (2-3)^2 + (3-4)^2 + (4-7)^2 + (5-2)^2 \; (6-5)^2 + (7-6)^2 + 0 + 0$$

$$= 1 + 1 + 9 + 9 + 1 + 1 = 22$$

From Bradley (1968, Table 1) we obtain:

Prob $(D \leq 22 \mid n = 9) <$ Prob $(D \leq 26 \mid n = 9) \leq 1\%$

(D) Finally we constructed a number of random variables whose distribution is independent of the value of λ, λ being the constant in the relationship

Poisson Parameter = λ * (oil produced)

Such a random variable is, for instance,

T = (sum of last m x_i's)/(sum of first m x_i's)

Where $2 \leq m \leq n/2$ (n being the number of our observations).

Then:

$$\text{Prob } (T \geq r) = \sum_{k=0}^{m-1} \binom{m+k-1}{k} r^k/(1+r)^{k+m}$$

For our data with n = 9 and m = 4 we compute t value of 3.77 and Prob $(T \geq 3.77)$ = 3.9%

If we extrapolate a production level for 1980, if we use the fact that there have been no spills in 1980, and if we take the oil produced since the last accident as a lower estimate for x_{10}, then we compute that t > 3.85 with n = 10 and m = 5.

For these values:

Prob (T > 3.85) = 2.23%

There are of course other random variables like T that one can construct and test, such as:

$$T_1 = (\max \frac{j\, x_j}{x_1 + x_2 + \ldots + x_j},\ j \geq n/2), \text{ and}$$

$$T_2 = (\max (x_{k+1}, \ldots, x_n)/\max (x_1, x_2, \ldots, x_k),\ k = \frac{n}{2}$$

These random variables also have distributions independent of λ, and when used, they indicate low likelihood that the X_i's are independent and identically distributed exponential random variables.

Thus it seems that there are excellent reasons to reject the hypothesis that the random variables X_1, X_2, ... X_n are identically distributed and independent. In what follows we will investigate whether we can construct a better model by assuming that the risk factor λ is not constant.

PART I

Let us assume that the exposure coefficient for the year 1964+i is $\lambda_i = \lambda f_i(p)$. Then we will seek to maximize the likelihood function:

$$\prod_{i=0}^{k} \frac{1}{n_i!} (\lambda_i t_i)^{n_i} \; e^{-\lambda_i t_i} = \lambda^n \; e^{-\lambda \Sigma f_i t_i} \prod_{i=0}^{k} \frac{(f_i t_i)^{n_i}}{n_i!}$$

Where n_i = number of spills in 1964+i,

t_i = oil production in 1964+i,

and $n = \Sigma n_i$

We remark that no matter what the value of p, in order to maximize the likelihood function we must choose:

$$\lambda = n/\Sigma f_i t_i$$

But if λ is so chosen, then the function to maximize becomes:

$$\Pi (f_i t_i)^{n_i} / (\Sigma f_i t_i)^n$$

The logarithmic derivative of the last expression is:

$$\Sigma \frac{n_i \; f_i'}{f_i} - n \frac{\Sigma f_i' t_i}{\Sigma f_i t_i}$$

If we try to maximize the likelihood function assuming that:

$f_i = 1.0$ or $f_i = 1/1+ip$ or $f_i = 1-ip$ or $f_i = p^i$,

then we obtain the following values for λ,p, and the likelihood function:

Function Used	λ	p	Likelihood Function
$f_i = 1.0$	2.05	N.A.	1.72 10^{-8}
$f_i = 1/1+ip$	16.14	1.402	4.74 10^{-7}
$f_i = 1-ip$	3.75	0.055	7.70 10^{-8}
$f_i = p^i$	7.33	0.827	2.10 10^{-7}

where λ is measured in spills per billions of barrels produced and represents the exposure coefficient in year 1964.

Following Edwards (1972), we will assume that the introduction of an extra parameter is justified if the natural logarithm of the likelihood function increases by two or more. In our case, for $f_i = 1/1+ip$ we obtain ℓn (47.4/2.72) = 3.32. Thus we get a significantly better fit by assuming that $f_i = 1/1+ip$.

This being said, there is another question to be answered:

The functional form of f_i was in each instance chosen in such a way that:

a. f_i is a "simple" function of i and p.

b. There is a set of values for p, S(f), such that:

$f_i(p) > f_{i+1}(p)$ for all $i = 0,1,2\ldots$ and p in S(f).

c. We had good reasons to believe that the value of p that maximizes the likelihood function would fall in S(f).

Is it possible that we can do significantly better by choosing some other function f that will satisfy conditions b and c?

To answer these questions we maximized the likelihood function by imposing no restriction other than that:

$$\lambda_0 \geq \lambda_1 \geq \lambda_2 \geq \cdots \qquad\qquad \geq \lambda_i \geq \lambda_{i+1} \geq \cdots$$

$$\text{(equivalently } f_0(p) \geq f_1(p) \geq f_2(p) > \cdots f_i(p) \geq f_{i+1}(p) \geq \cdots)$$

Then, we found that the maximum value of the likelihood function is $2.96\ 10^{-6}$. This is better than $4.74\ 10^{-7}$ but not significantly so if we follow Edwards' rule of thumb (Edwards, 1972). Indeed, $\ell n(29.6/4.74) = 1.83 < 2.0$. On the other hand we will have to:

a. Make heroic assumptions on the functional form of f in order to achieve or even approximate λ_0, λ_1, ..

b. Use the data to determine the functional form of f as well as the value of p, thus losing another degree of freedom.

We can therefore assume that the choice of $f_i = 1/1+ip$ is a reasonably good one.

In Table III we show the values of $\lambda_i = \lambda/(1+ip)$ which were found by maximizing the likelihood function (i.e., by setting $\lambda = 16.14$ and $p = 1.402$). Notice that λ_i is the expected number of spills per billion barrels produced in 1964+i.

We notice that if we readjust the oil produced in (1964+i) by the factor 1/(1+ip), then we should have a Poisson process with $\lambda = \lambda_0$. We can therefore apply the same battery of tests we used in Part I to this new hypothesis and we will do so in the next section.

PART II

In this section we will examine whether the model proposed in Part I is free of the shortcomings that the original model exhibits. We will also examine whether the model's estimates of λ's values display any stability when a number of years are removed from the record.

The adjusted amounts of oil produced between two successive accidents are:

31.3 55.9 59.1 151.5 5.4 33.1 29.0 70.6 and 120.5

These adjusted amounts were found by adjusting each year's production by a factor ($1/1+ip$ for year $1964+i$, $i = 1, 2, \ldots 15$) and then by interpolating.

If our model is correct, then these values are realizations of independent and identically distributed exponential random variables.

Thus we have:

(A) The observed run of signs is:

+ + + - + - + +

and

Prob (5 or less runs $| m = 9$) = 43.47%, (Bradley, 1968, Table X), while

Prob (longest run $\geq 3 \mid n = 9$) = 38.15%, (Olmstead, 1964, Table 2).

(B) Kendall's test.

We have:

$$I = 2 + 3 + 3 + 5 + 0 + 1 + 0 + 0 = 14$$

$$T = 6 + 4 + 3 + 0 + 4 + 2 + 2 + 1 = 22$$

$$S = T - I = 8$$

From Edward's (1972, Table I) we obtain:

$$\text{Prob } (S \geq 8 \mid n = 9) = 23.8$$

While

$$\text{Prob } (|S| \leq 8 \mid n = 9) = 1 - 2 * \text{Prob } (S \geq 10 \mid n = 9) =$$

$$1 - 2 * 0.179 = 1 - 0.358 = 64.2\%$$

(C) Hotelling and Pabst's test.

$$D = \Sigma[i - \text{rank } (x_i)]^2 = (1-3)^2 + (2-5)^2 + (3-6)^2 + (4-9)^2 + (5-1)^2 + (6-4)^2 + (7-2)^2 + (8-7)^2 + (9-8)^2 = 94$$

From Bradley (1968, Table I) we obtain:

$$\text{Prob } (D \leq 62 \mid n = 9) = \text{Prob } (D \geq \tfrac{1}{3}n\ (n^2 - 1) - 62 \mid n = 9) \leq 10\%$$

For: $n = 9$ $\frac{1}{3}\, n\ (n^2 - 1) = \frac{1}{3}\ 9\ 80 = 240$

Thus:

$$\text{Prob } (62 \leq D \leq 178) \geq 80\%,$$ and 94 is in the 80% central range.

(D) (Sum of last four/sum of first four) = 253.2/297.8 = 0.85

and

$$\text{Prob } (T \geq 0.85 \mid m = 4) = 58.80\%$$

We see then, that the tests which indicated that there was something wrong with our first model do indicate that there is better support for the hypothesis of a decreasing λ (increased safety).

Finally, in order to see how stable are our estimates of λ, we put on the same graph (Graph I) the following information:

(1) The estimated risk factor (expected number of spills per billion barrels produced) for years 1966 to 1979 when the risk factor λ_i is computed as $\lambda/1+ip$ with $\lambda = 16.4$ and $p = 1.4$. These points are represented by ●.

(2) The risk factor for year 1964+i that we would have estimated, if, we had followed the same procedure as in I, when data only for years 1964 through 1964+i were available. These points are represented by a ▲ .

(3) The risk factor we would have estimated if we assumed no improvement and data for years 1964 to 1964+i only, i $\geq$, were available.

These points are represented by ■ .

The graph shows that there is little difference between the " ● " points and the ▲ points from 1971 onwards.

PART III

Let us assume that the process which we have been observing is a Poisson process for which the exposure variable in year 1964+i was:

$$\lambda^t{}_i = \lambda^t/(1+ip), \ i = 0,1, \ldots., 15$$

From the likelihood function for the observed record we obtained the estimates $\hat{\lambda} = 16.14$ and $\hat{p} = 1.4$. It is, of course, evident that a different realization of the same process would lead us to a different estimator of (λ^t, p^t). We are therefore interested not only in the estimator $(\hat{\lambda}, \hat{p})$, but also in knowing how good an estimator $(\hat{\lambda}, \hat{p})$ is. More important, since we plan to use $\hat{\lambda}$ and $\hat{p}$ to estimate $\lambda^t{}_{15}$, the exposure coefficient for 1979 and beyond, we would like to have an idea of how $\hat{\lambda}_{15} = \hat{\lambda}/1{+}15p$ is distributed around $\lambda^t{}_{15}$.

Since it is quite unlikely that we can find an analytic expression for the distribution of $\hat{\lambda}_{15}$, we proceeded as follows:

A. When we find $\hat{\lambda}$ and $\hat{p}$ by maximizing $L(\lambda,p)$, L being the likelihood function, we can also find a region D around $(\hat{\lambda}, \hat{p})$ such that:

$$D = [(\lambda,p) \quad L(\lambda,p) \geq e^{-2} L(\lambda,p)]$$

and call it the 2-unit support region for $(\hat{\lambda}, \hat{p})$. Thus we can obtain a measure of how good an estimate of (λ^t, p^t) we obtain by $(\hat{\lambda}, \hat{p})$ (Edwards, 1972).

Since our likelihood function is of the form:

$$L\ (\lambda,p) = A(p)\ \lambda^n e^{-\ \lambda B(p)},\ A(p),\ B(p) \geq 0,$$

then for a fixed p, L is an increasing function of λ while $o \leq \lambda \leq n/B(p)$, and a decreasing function thereafter. Thus L, viewed as a function of λ, attains its maximum at $\lambda = n/B(p)$, and the said maximum is $\frac{A(p)}{B^n(p)} n^n e^{-n}$. Thus in order to find D, we first solved the equation:

$$\frac{A(p)}{B^n(p)}\ n^n e^{-n} = L\ (\frac{n}{B(p)},\ p) = e^{-2}\ L\ (\hat{\lambda},\hat{p})$$

which has two solutions, $p_1 = 0.08$ and $p_2 = 7.23$. For p outside $[p_1,\ p_2]$, the inequality $L(\lambda,p) \geq e^{-2}\ L(\hat{\lambda},\hat{p})$ has no solutions in λ. On the other hand if p is in $[p_1,\ p_2]$, there are values $\lambda min(p)$ and $\lambda max(p)$ such that:

$$[L\ (\lambda,p) \geq e^{-2}\ L\ (\hat{\lambda},\hat{p})] \leftrightarrow [\lambda min(p) \leq \lambda \leq \lambda max(p)]$$

Between these values lies $\lambda opt(p) = n/B(p)$, the value that maximizes $L(\lambda,p)$, viewed as a function of λ. In Table IV we give the values of $\lambda min(p)$, $\lambda opt(p)$ and $\lambda max(p)$ for different values of p. We also give $*\lambda(p) = \lambda min(p)/1+15p$, $\bar{\lambda}(p) - \lambda opt(p)/1+15p$, and $\lambda*(p) = \lambda max(p)/1+15p$, which are the exposure rates for 1979 that correspond to the λ values in columns 2-4.

The values of λmin, λopt and λmax are depicted in Graph II. The values of $*\lambda, \bar{\lambda}$ and $\lambda*$ are depicted in Graph III.

Finally let us remark that, as the graphs imply, λopt is an increasing function of p, while $\bar{\lambda}$ is a decreasing one.

Indeed:

$$\lambda opt(p) = \frac{n}{\sum_{1=0}^{15} \frac{ti}{1+ip}}, \text{ while}$$

$$\bar{\lambda}(p) = \lambda opt(p) \frac{1}{1+15p} = \frac{n}{\Sigma t_i \left(\frac{15}{i} - \frac{15 - i}{i(1+ip)}\right)}$$

Evidently the denominator of λopt is a decreasing function of p, while the denominator of $\bar{\lambda}$ is an increasing function of p.

B. For a given pair (λ,p) we can simulate a number of observed records, consistent with the actual volume of oil produced. For each such record r we can derive the maximum likelihood estimates $\hat{\lambda}(r)$ and $\hat{p}(r)$. Then we can compare $\hat{\lambda}_{15}(r) = \hat{\lambda}(r)/1+15p(r)$ to $\hat{\lambda}_{15} = \hat{\lambda}/1+15\hat{p}$, where $\hat{\lambda}$ and $\hat{p}$ are the maximum likelihood estimates for the actually observed record. More specifically: Let $\lambda_{15} = \lambda/1+15p$ be the "true" exposure rate in 1981, and let $\hat{\lambda}_{15} = \hat{\lambda}/1+15\hat{p}$ be the rate we estimated via the observed record. Then we call tail of $\hat{\lambda}_{15}$ the interval $(-\infty, \hat{\lambda}_{15})$ if $\hat{\lambda}_{15} < \lambda_{15}$ and $(\hat{\lambda}_{15}, +\infty)$ if $\hat{\lambda}_{15} > \lambda_{15}$. For a set of simulated records we will count then the number of instances in which $\hat{\lambda}_{15}(r) = \hat{\lambda}(r)/1+15\hat{p}(r)$ is in the tail of $\hat{\lambda}_{15}$ as well as the number of instances in which $\hat{\lambda}_{15}(r)$ is in the tail of $\hat{\lambda}_{15}$ and (λ,p) is <u>not</u> within the 2-unit support of $(\hat{\lambda}(r), \hat{p}(r))$. The results of these simulations are given in Table V. Each entry of the table is of the form a/b where a is the instances in $\hat{\lambda}_{15}(r)$ was in the tail of $\hat{\lambda}_{15}$, and b the cases in which $\hat{\lambda}_{15}(r)$ was in the tail of $\hat{\lambda}_{15}$ and (λ,p) was not in the 2-unit support of $(\hat{\lambda}(r),\hat{p}(r))$.

CONCLUSION

On the basis of the available evidence, it seems reasonable to assume:

a. That there has been an improvement in the safety record.

b. That the risk factor in 1979 was 0.73 spills per billion barrels produced and not 2.05 as we would have computed it, if we were to assume a Poisson process and constant λ.

It is conceivable that new data could make us adopt a different model. One possibility is that there has been an abrupt change and that a good model could be one that would postulate no gradual change in λ but would instead discard years 1964 and 1965 from our data base. We want to stress here that there is not, presently, sufficient statistical support that would make this model preferable to the one we propose. Furthermore, it can be readily seen that this model's estimate for year 1979 (1.46) would be substantially lower than 2.05 although not as low as 0.73.

Finally we should observe that however we might modify our conclusions, as new data is collected, it is quite unlikely that the model with constant λ - implying no increase in safety - will be shown to be correct.

TABLE I

U.S. Offshore Oil Production, 1964-80

YEARS	PRODUCTION (in millions of barrels)
1964	114.977
1965	136.236
1966	175.187
1967	205.861
1968	252.016
1969	295.429
1970	337.123
1971	390.180
1972	378.497
1973	360.899
1974	322.354
1975	303.159
1976	294.183
1977	276.984
1978	270.421
1979	262.275
1980	256.000*

* Extrapolated from the production in previous years.

Source: U.S. Geological Survey, 1980.

TABLE II

Accidents in which more than 1,000 barrels were spilled, and the estimated amount of oil produced between the previous accident and the present one. (For the the first accident, the amount given is the estimated production between January 1, 1964 and the day of the accident.)

SPILL	DATE (month/day/year)	OIL PRODUCED (millions of barrels)
1	4/8/64	31.3
2	10/3/64	55.9
3	7/9/65	103.1
4	1/23/69	712.9
5	3/16/69	43.5
6	2/10/70	270.5
7	12/1/70	272.5
8	1/9/73	804.9
9	11/23/79	2054.3

Source for spills and dates: U.S. Geological Survey, 1980

TABLE III

The computed values of exposure coefficient for the year 1964+i, $\lambda_i = \lambda/1+ip$ when $\lambda = 16.14$ and $p = 1.402$ are:

YEAR	VALUE OF λ (spills/million barrels)
1964	16.14
1965	6.72
1966	4.24
1967	3.10
1968	2.49
1969	2.05
1970	1.72
1971	1.49
1972	1.32
1973	1.19
1974	1.07
1975	0.98
1976	0.91
1977	0.84
1978	0.78
1979	0.73

TABLE IV

Ranges of λ for different values of p.

	Calculations of λ			Exposure rates for 1979		
p	λmin	λopt	λmax	*λ	λ_{15}	λ*
0.08	3.29	3.29	3.29	1.50	1.50	1.50
0.09	2.94	3.41	3.92	1.25	1.45	1.67
0.10	2.88	3.55	4.31	1.15	1.42	1.72
0.20	3.06	4.83	7.18	0.77	1.21	1.79
0.25	3.25	5.43	8.43	0.68	1.14	1.77
0.50	4.24	8.18	14.02	0.50	0.96	1.65
0.75	5.21	10.63	18.92	0.43	0.87	1.54
1.00	6.15	12.87	23.29	0.38	0.80	1.46
1.25	7.07	14.95	27.23	0.36	0.76	1.38
1.50	7.98	16.88	30.77	0.34	0.72	1.31
1.75	8.88	18.70	33.95	0.33	0.69	1.25
2.00	9.78	20.40	36.81	0.32	0.66	1.19
2.25	10.69	22.00	39.38	0.31	0.63	1.13
2.50	11.60	23.51	41.67	0.30	0.61	1.08
2.75	12.52	24.94	43.71	0.30	0.59	1.03
3.00	13.45	26.30	45.51	0.29	0.57	0.99
3.25	14.39	27.59	47.10	0.29	0.55	0.95
3.50	15.36	28.81	48.47	0.29	0.54	0.91
3.75	16.35	29.98	49.65	0.29	0.52	0.87
4.00	17.37	31.09	50.64	0.28	0.51	0.83
4.25	18.41	32.15	51.45	0.28	0.50	0.79
4.50	19.49	33.17	52.09	0.28	0.48	0.76
4.75	20.62	34.14	52.57	0.29	0.47	0.73
5.00	21.79	35.07	52.87	0.29	0.46	0.70
5.25	23.03	35.96	53.00	0.29	0.45	0.66
5.50	24.33	36.81	52.97	0.29	0.44	0.63
5.75	25.72	37.63	52.75	0.29	0.43	0.60
6.00	27.23	38.42	52.33	0.30	0.42	0.58
6.25	28.88	39.18	51.68	0.30	0.41	0.55
6.50	30.74	39.91	50.74	0.31	0.41	0.52
6.75	32.93	40.61	49.40	0.32	0.40	0.48
7.00	35.77	41.29	47.34	0.34	0.39	0.45
7.10	37.33	41.55	46.08	0.35	0.39	0.43
7.20	39.71	41.81	43.98	0.36	0.38	0.40
7.21	40.11	41.84	43.62	0.37	0.38	0.40
7.22	40.61	41.86	43.14	0.37	0.38	0.39
7.23	41.54	41.89	42.23	0.38	0.38	0.39

TABLE V

Simulation of observed records for different combinations of p and λ.

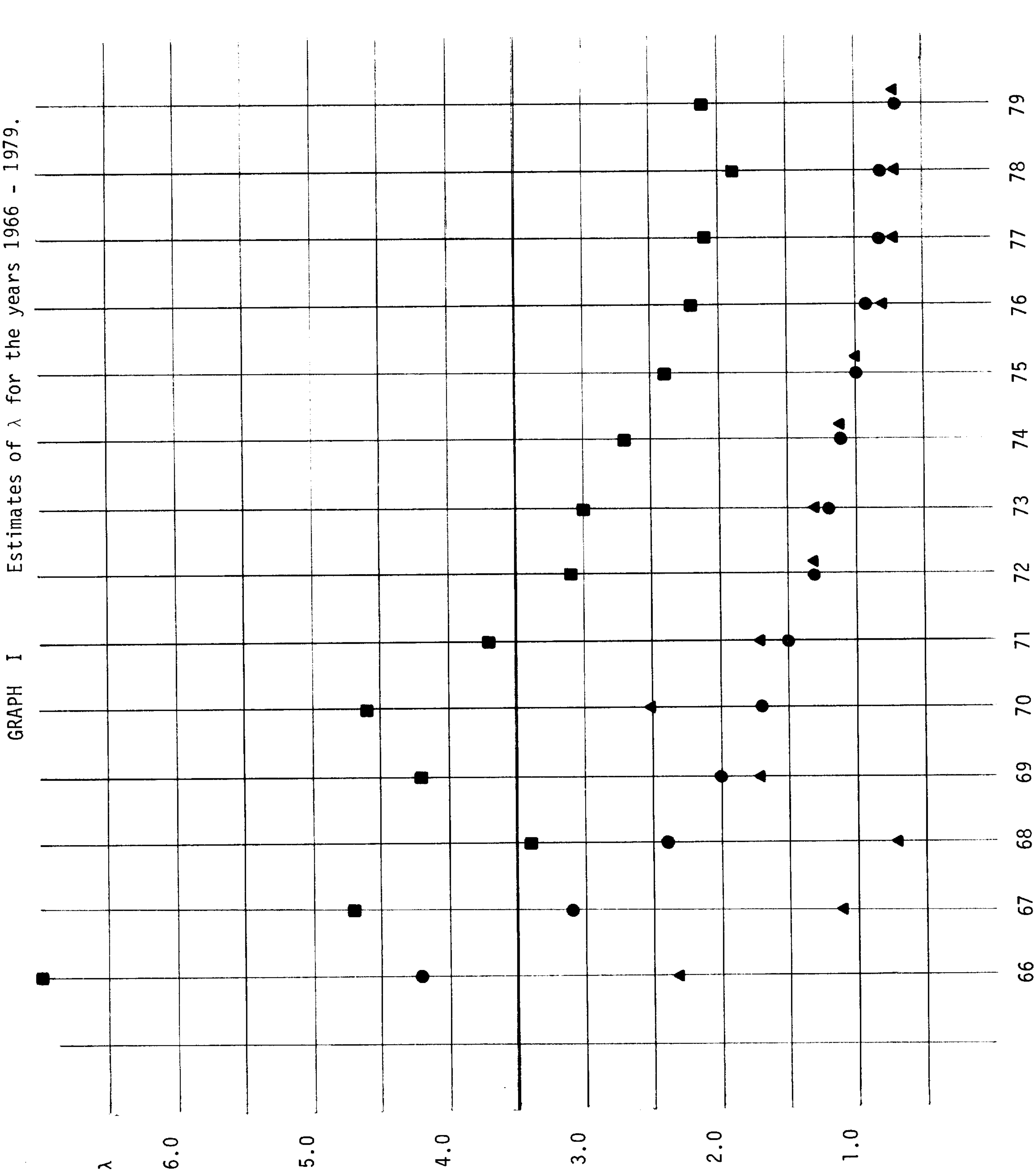
p	λ	λ_{15} \ N	100	200	300	400
0.08	3.29	1.50	5/1	7/2	13/2	38/4
0.10	2.88	1.15	18/0	45/0	63/2	87/3
0.10	3.55	1.42	9/1	16/1	30/3	44/6
0.10	4.31	1.72	4/0	8/2	10/2	16/3
0.20	3.06	0.76	41/1	77/3	130/7	167/9
0.20	4.83	1.21	7/0	18/0	43/2	61/2
0.20	7.18	1.79	2/0	6/1	7/2	10/1
0.50	4.24	0.50	27/2	54/4	87/7	127/10
0.50	8.18	0.96	27/1	49/2	75/3	93/3
0.50	14.02	1.65	2/1	2/1	2/1	3/1
1.50	7.98	0.34	14/3	23/3	44/5	51/6
1.50	16.88	0.72	49/0	107/2	152/3	205/5
1.50	30.77	1.31	9/0	17/0	24/1	27/1
7.23	41.88	0.38	11/4	20/5	23/5	28/5
0.00	2.05	2.05	4/0	10/3	19/3	24/3
0.00	2.00	2.00	6/0	14/1	23/2	31/3

Each entry k/m shows that in k instances $\hat{\lambda}_{15}(r)$ was in the tail of $\hat{\lambda}_{15}$ and that in m out of these k instances (λ,p) was not in the 2-unit support of $(\hat{\lambda}(r), \hat{p}(r))$.

<u>NOTE</u> that the maximum likelihood estimates are: $\hat{\lambda} = 16.14$, $\hat{p} = 1.40$, and $\hat{\lambda}_{15} = 0.73$

GRAPH I

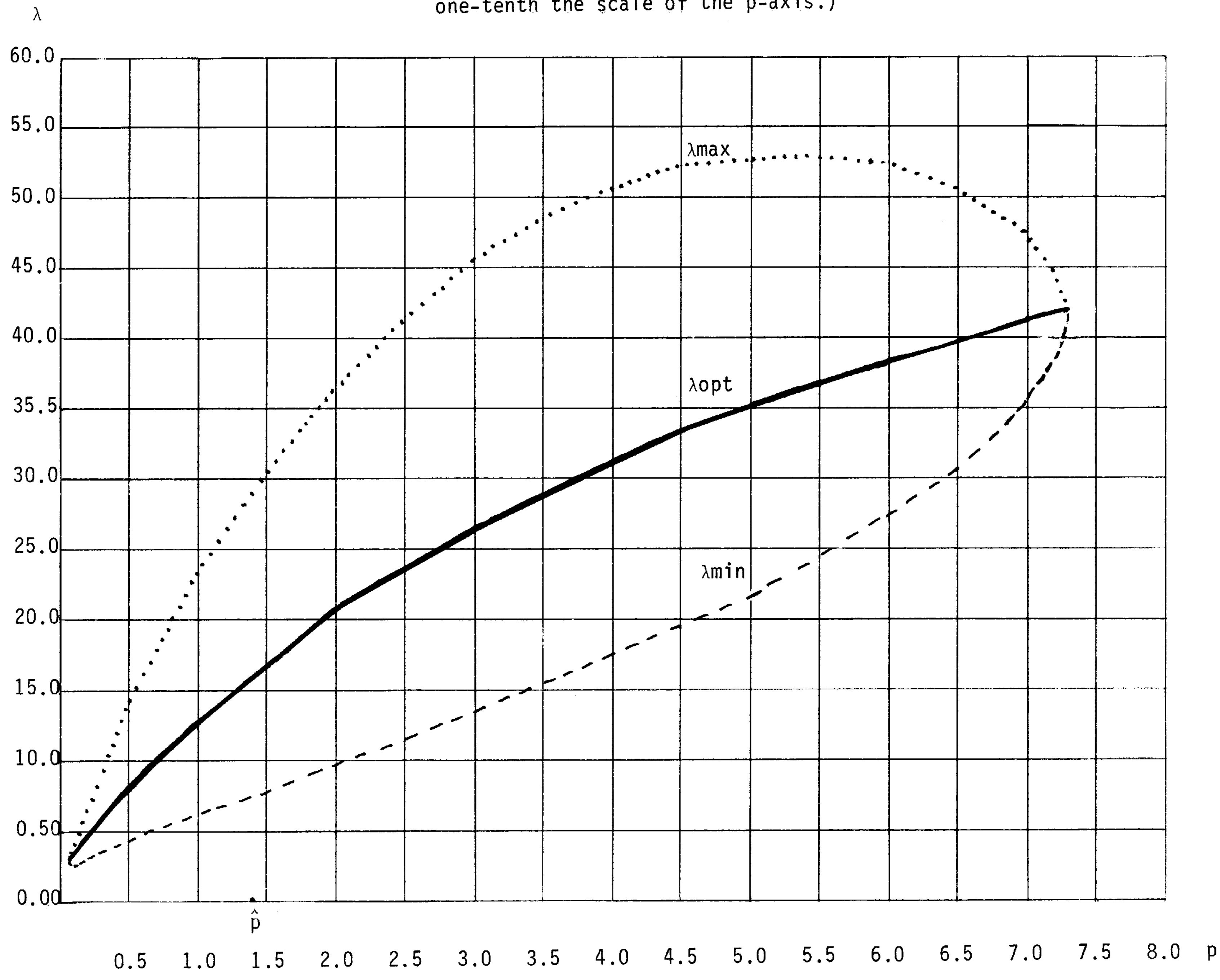
[Symbol explanation on page 13.]



GRAPH II

Values of λmin, λopt and λmax vs p

(Notice that for graphing purposes the scale on the λ-axis is one-tenth the scale of the p-axis.)

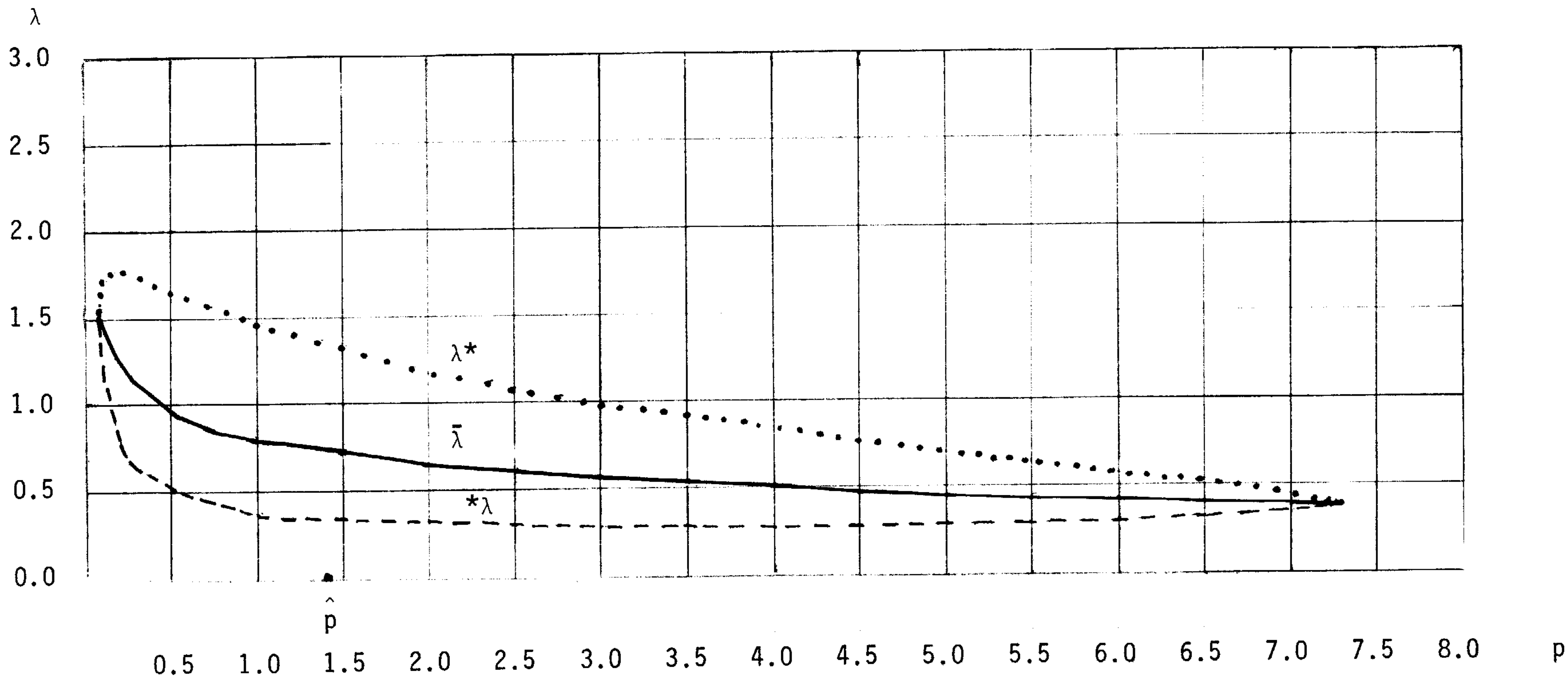


GRAPH III

Values of $*\lambda$, $\bar{\lambda}$ and λ^* vs p

(λ^* has a local max near p = 0.2

$*\lambda$ has a local min near 4.25)



REFERENCES

Bradley, James V., 1968, Distribution free statistical tests: Prentice Hall

Cox, D.R. and Lewis, P.A.W., 1968, The statistical analysis of series of events: Barnes and Noble

Edwards, A.W.F., 1972, Likelihood: Cambridge University Press

Kendall, Maurice G., 1955, The measurement of rank correlation: London Griffin

Olmstead, P.S., 1964, Distribution of sample arrangements for runs-up and down: Annuals of Mathematical Statistics, v. 17, pp. 24-33.

U.S. Geological Survey, Convervation Division, June 1980, Outer Continental Shelf Statistics.